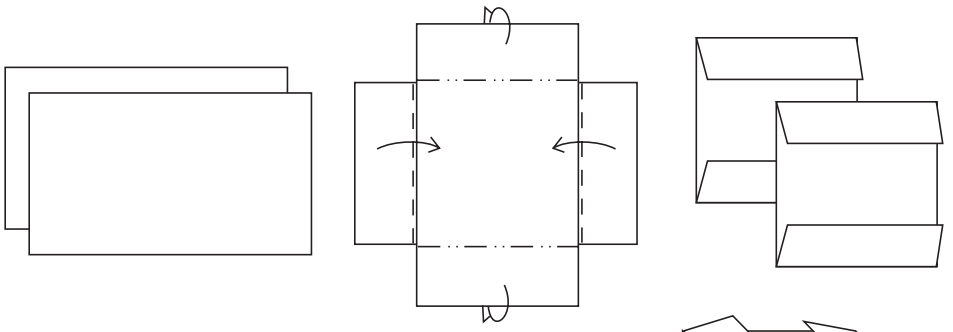
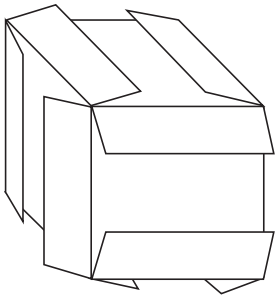
Business Card Cubes and the Menger Sponge

One of the easiest modular origami things to make from standard business cards is a cube. It takes 6 cards. To make a unit, make a "plus" sign with two cards and bend them around each other. Separate them, and you'll have just made two units!



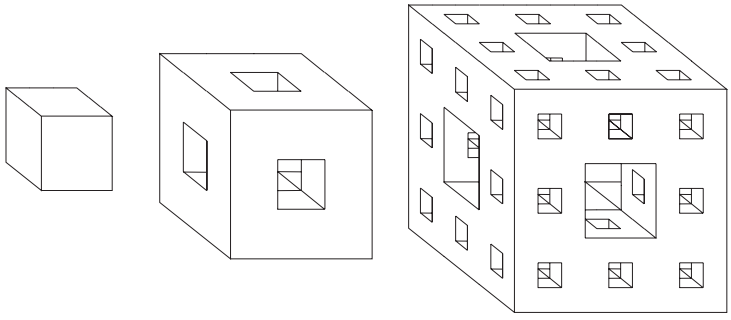
Make six units and use them to form a cube. Each unit is a face of the cube, and the folded flaps have to grip the other units. When you're done, you'll still see these folded flaps on the outside, gripping it all together.



It's possible to take 6 more units and use them to "panel" the cube so that its faces are smooth. Do you see how this would work?

Two (unpaneled) cubes can be locked together along a face by making the folded flaps grip into each other. This allows you to build structures with these cubes.

Activity: Working in groups, make a "Level 1" **Menger Sponge**. A Menger Sponge is a fractal object made by starting with a cube (Level 0), then taking 20 cubes and making a cube frame with them (Level 1), and then taking 20 of these frames and making a bigger cube frame with them (Level 2), and so on. If we scale the model down after each iteration (so it remains the same size throughout), in the infinite case we'll get what is known as Menger's Sponge.



How many business cards will it take to make a Level 1 Sponge? With paneling?

Question 1: Let U_n = the number of business cards needed to make an unpaneled Level n Menger's Sponge. So $U_0 = 6$.

Compute values for U_1, U_2, and U_3. Find a closed formula for U_n in terms of n.

Question 2: Let P_n = the number of business cards needed to make a **paneled** Level n Menger's Sponge. So $P_0 = 12$.

Find P_1, P_2, and P_3. Can you find a formula (not necessarily closed) for P_n in general? How about a closed formula?